Further Maths Revision Paper 5

This paper consists of 5 questions covering CP1, CP2, FP1 and FM1.
(AS Further Maths: Q4 and Q5)
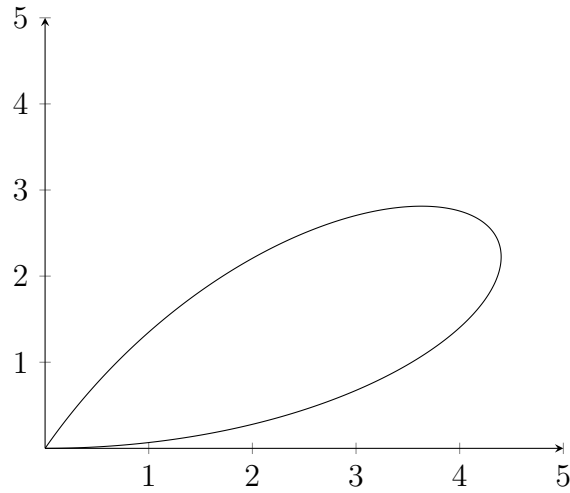
1

Figure 1

Figure 1 shows a section of the graph $r = 5 \sin 3\theta$.
Find the area enclosed by the loop.

2

$$y = (1 + x^4)\sin x$$

Show that

$$\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} = (x^4 - 72x^2 + 25)\sin x - 16x(x^2 - 6)\cos x$$

3

A car of mass 1200kg tows another car of mass 800kg, the frictional resistances being 120N and 80N respectively.
If the tow rope has a breaking tension of 2000N find:

(a) the maximum possible acceleration.

(b) the maximum power the towing car can use at the instant when the speed is 10km/h

4

Given the differential equation
$$100\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 1 + (y-3)^2$$
with conditions $y = 4$ when $x = 0$ and $y = 4$ when $x = 1$
Use the approximation
$$y_{r+1} \approx 2y_r - y_{r-1} + h^2 \left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)_r$$
with $h = 1$ to find the value of y when $x = 4$

5

(a) Show that $\alpha = 3 + 2i$ is a root of $z^3 - 2z^2 - 11z + 52 = 0$.

(b) Hence find all the solutions of $z^3 - 2z^2 - 11z + 52 = 0$